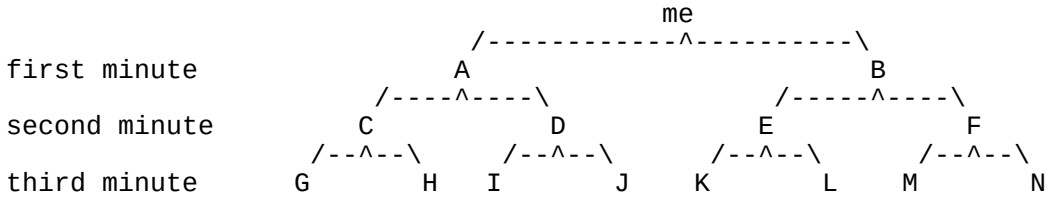
Name:_____**Date:**_____

Telephone Trees

This problem is about the best way to pass on news to lots of people using the telephone. We could just phone everyone ourselves, so 14 people to share the news with would take 14 separate calls. Suppose each call takes just 1 minute, then we will be on the phone at least 14 minutes (if everyone answers their phone immediately).

Can we do better than this? We could use the speakers on the phone - the "hands free" facility which puts the sound out on a speaker rather than through the handset so that others in the room can hear the call too. For the sake of a puzzle, let's suppose that 2 people hear each call. That would halve the number of calls I need to make. My 14 calls now reduces to 7.

Can we do better still?
Well, we could ask each person who receives a call to not only put the call through the loudspeakers but also to do some phoning too. So if two people hear the message, they could each phone two others and pass it on *in the same way* and so on. Here's what it looks like if I have 14 people to phone in this system as the calls "cascade". In the first minute, my first call is heard by A and B. A's call is heard by both C and D; B's call by E and F, and so on as in this diagram:



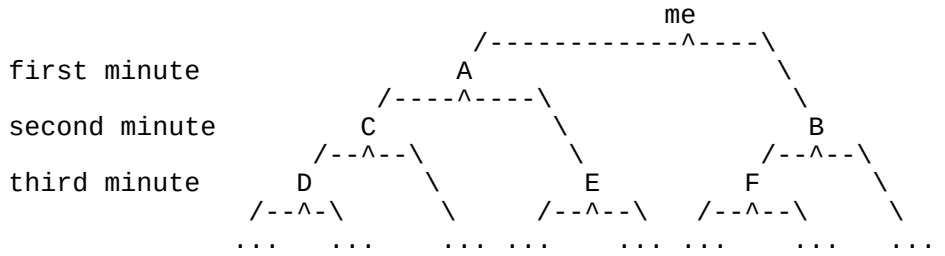
So all 14 people have heard the news in only 3 minutes! [This is an example of *recursion* - applying the same optimizing principle at *all* levels of a problem.]

Can we do even better than this?
Yes - if all the people got together in one room, it would only take one minute! So let's assume that I cannot get everyone together and I have to use the phone.

Now here is your puzzle. The phones in my company are rather old and do not have an external speaker (and no "conference call" facility) - only one person can hear each call. So I decide that I will phone only **two** people using two separate calls. I shall give them the news and then ask that they *do the same* and phone just **two** more people only. What is the shortest time that the news can pass to 14 people?

Name:_____ **Telephone Trees**

1. Draw the *cascade tree* of telephone calls, or the *telephone tree* for this problem. It begins like this:



   How does the tree continue?
2. What is the maximum number of people in the office that could hear the news within N minutes using this method?
   Why is the answer related to the Fibonacci numbers?

From Ron Knott's Fibonacci Numbers and the Golden Section web site.